Ex: Find the value of $f(1.5)$ and $f(7.5)$ from the given table:

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	256	343	512

Ans: To compute $f(1.5)$, we use Newton's forward formula, since $x = 1.5$ is near the beginning of the table.

forward

$$f(x) \approx y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots + \frac{u(u-1)\cdots(u-n+1)}{n!}\Delta^n y_0$$

Here $x = 1.5$, $x_0 = 1$, $h = 1$; $u = x - x_0/h = 0.5$

Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1				
		7			
2	8		12		
		19		6	
3	27		18		0
		37		6	
4	64		24		0
		61		6	
5	125		30		0
		91		6	
6	216		36		
		127		6	
7	343		42		
		169			
8	512				

$$f(1.5) = 1 + (0.5)7 + (0.5)(-0.5)\left(\frac{12}{2!}\right) + (0.5)(-0.5)(-1.5)\left(\frac{6}{3!}\right)$$

$$= 3.375$$

To compute $f(7.5)$, we use Newton's Backward formula, since 7.5 is near end of table.

$$f(x) = y_n + u\Delta y_{n-1} + \frac{u(u+1)}{2}\Delta^2 y_{n-2} + \cdots$$

$x = 7.5$, $x_n = 8$, $h = 1$, $u = \frac{x - x_n}{h} = -0.5$

$$f(7.5) = 512 + (-0.5)169 + (-0.5)(0.5)\left(\frac{42}{2!}\right) + (-0.5)(0.5)(1.5)\left(\frac{6}{3!}\right) = 421.875$$

H.W Given the following table

x :	0	5	10	15	20
$f(x)$:	1	1.6	3.8	8.2	15.4

Construct the difference table and compute $f(21)$ by Newton's Backward formula.

Ans: class work (Example of extrapolation)

H.W:- Calculate from the following table the value of y when $x = 1.6$

x	1	1.5	2	2.5	3
$f(x)$	0.11246	0.14032	0.168	0.19547	0.2227

H.W. Find the cubic polynomial which takes the values:

(Take home Surprise Test)

$y(1) = 24$

$y(3) = 120$

$y(5) = 336$

$y(7) = 720$

Hint

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	24			
		96		
3	120		120	
		216		48
5	336		168	
		384		
7	720			

Hence, or otherwise, obtain the value of $y(8)$.

(Time Limit for submission : Today upto 11:59 PM)

Hint

Newton's forward formula :

$$f(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

Here,

$$x_0 = 1,\ h = 2,\quad u = \frac{x-1}{2}$$

$$f(x) = .24 + \left(\frac{x-1}{2}\right)96 + \cdots$$ (Do it yourself!)

It. W.r Values of x (in degrees) and $\sin x$ are given in the following table:

x :	15	20	25	30	35	40
$f(x)$:	0.258	0.3420	0.4224	0.5	0.573	0.642

Determine the value of $\sin 38^\circ$

Ans : 0.6156614

Central Difference Interpolation

Newton's forward or backward formulaes are useful only to interpolate near the beginning, or the end of the table. So, interpolating near the middle of the tabular values through these methods give error.. There are some other interpolation method (Gauss, Stirling's & Bessel's interpolation) which gives more accurate result near middle of table.

* Find the missing value in the following table :

x	1	2	3	4	5
$f(x)$	9	-	15	23	39

Explain why it differs from $2^x + 7 = 11$

Assume $y = f(x)$ be a polynomial of deg 3

[∵ only 4 values given]

$$\Delta^4 y_0 = 0$$

$$\Rightarrow (E-1)^4 y_0 = 0$$

$$\Rightarrow \left(E^4 - 4E^3 + 6E^2 - 4E + 1\right) y_0 = 0$$

$$\Rightarrow y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$\Rightarrow 39 - 4(23) + 6(15) - 4y_1 + 9 = 0$$

$$\Rightarrow y_1 = 11.5$$

Result differs since we are interpolating exponential function with polynomial.

H.W. Find the missing values :

x	0	5	10	15	20	25	30
$f(x)$	1	3	–	73	225	–	1153

Hint : Think about the terms $\Delta^5 y_0$, $\Delta^5 y_1$

Gauss's Forward Interpolation Formula